Hybrid solution for transient internal convection with axial diffusion

Integral transforms and local instantaneous filtering

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Abstract

Purpose – This paper seeks to analyze transient convection-diffusion by employing the generalized integral transform technique (GITT) combined with an arbitrary transient filtering solution, aimed at enhancing the convergence behavior of the associated eigenfunction expansions. The idea is to consider analytical approximations of the original problem as filtering solutions, defined within specific ranges of the time variable, which act diminishing the importance of the source terms in the original formulation and yielding a filtered problem for which the integral transformation procedure results in faster converging eigenfunction expansions. An analytical local instantaneous filtering is then more closely considered to offer a hybrid numerical-analytical solution scheme for linear or nonlinear convection-diffusion problems.

Design/methodology/approach – The approach is illustrated for a test-case related to transient laminar convection within a parallel-plates channel with axial diffusion effects.

Findings – The developing thermal problem is solved for the fully developed flow situation and a step change in inlet temperature. An analysis is performed on the variation of Peclet number, so as to investigate the importance of the axial heat or mass diffusion on convergence rates.

Originality/value – This paper succeeds in analyzing transient convection-diffusion via GITT, combined with an arbitrary transient filtering solution, aimed at enhancing the convergence behaviour of the associated eigenfunction expansions.

Keywords Convection, Transforms, Diffusion

Paper type Research paper

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Nomenclature

b	= half width of parallel plates channel (equation (12))	x, y	= dimensionless space variables (problem (13a)-(13g))
$d(\mathbf{x})$	= linear dissipation coefficient (equation (1a))	$\mathbf{x}$	= position vector
$f(\mathbf{x})$	= initial condition (equation (1b))	$w(\mathbf{x})$	= transient operator coefficient (equation (1a))
$k(\mathbf{x})$	= diffusion operator coefficient (equation (1a))	*Greek symbols*	
N_i	= normalization integral of the eigenvalue problem (equation (8))	α, β	= coefficients for the boundary condition (1c)
$P(\mathbf{x}, t, T)$	= nonlinear source term appearing in equation (1a)	ε	= error estimator (equation (11))
$T(\mathbf{x}, t)$	= potential	μ_i	= eigenvalues of problem (5a) and (5b)
t	= time variable	ψ_i	= eigenfunctions of problem (5a) and (5b)
u_0	= parameter appearing in equation (12)	$\phi(\mathbf{x}, t, T)$	= nonlinear source term appearing in equation (1c)
$u(y)$	= dimensionless velocity field (equation (12))		

Introduction

Along the last few decades, various purely discrete and semi-analytical approaches have been developed and applied to the approximate solution of diffusion and convection-diffusion problems. In parallel, a number of error control schemes have been proposed and tested to assess and/or to improve the accuracy of such approximate methodologies. While discrete numerical methods have been proved effective and flexible in handling different classes of heat and fluid flow problems, the automatic control and estimation of errors within the associated algorithms, in particular for multidimensional applications, still pose some numerical analysis difficulties, which are inherent to their discrete nature. In this context, a number of hybrid numerical-analytical methodologies have appeared in the open literature, that, to within different degrees of success, attempt to match the classical analytical ideas with the present knowledge basis on numerical analysis, in the search for more accurate, robust and economical options to the now well-established discrete solution methods.

Within the last two decades, the classical integral transform method was progressively generalized under a hybrid numerical-analytical concept (Cotta, 1993, 1994a, 1994b, 1998; Serfaty and Cotta, 1992; Cotta and Mikhailov, 1997). This approach now offers user-controlled accuracy and efficient computational performance for a wide variety of non-transformable problems, including the most usual nonlinear formulations in heat and fluid flow applications. Besides, being an alternative computational method in itself, this hybrid approach is particularly well suited for benchmarking purposes. In light of its automatic error-control feature, it retains the same characteristics of a purely analytical solution. In addition to the straightforward error control and estimation, an outstanding aspect of this method is the direct extension to multidimensional situations, with only a moderate increase in computational effort. Again, the hybrid nature is responsible for this behavior, since the analytical part in the solution procedure is employed over all but one independent variable, and the numerical task is always reduced to the integration of an ordinary differential system over this single independent variable.

This generalized integral transform technique (GITT) is now a well-established computational approach (Cotta, 1993, 1994a, 1994b, 1998; Serfaty and Cotta, 1992; Cotta and Mikhailov, 1997) and convergence enhancement strategies were proposed (Cotta and Mikhailov, 1997; Scofano Neto *et al.*, 1990; Leiroz and Cotta, 1990; Almeida and Cotta, 1996), aimed at reducing computational costs by offering more efficient eigenfunction expansions with lower truncation orders, for the same requested global accuracy. More recently, a local-instantaneous filtering strategy was proposed, Macedo *et al.* (1999), based on analytical filtering solutions, which present both space and time dependence, within ranges of the time numerical integration path. For instance, representative linearized or just simplified versions of the original problem in a certain time interval, after being exactly solved through the classical integral transform approach, more effectively partially filter the original problem source terms, which are responsible for deviating the convergence behavior from the spectral exponential pattern. Then, the filter can be automatically redefined for the next time variable range, by prescribing a desirable maximum value for the system truncation order, while still satisfying the user requested global accuracy target.

The previous experience with these different filtering strategies is now here recalled to propose a hybrid numerical-analytical integral transform procedure, which adopts a local instantaneous filter (LIF) for a typical transient convection-diffusion partial differential system. This work is aimed at complementing the analysis in Macedo *et al.* (1999) by introducing convective terms and examining the behavior of the proposed filtering for parabolic-hyperbolic formulations. An application modeled by a transient convection-diffusion formulation is then considered for illustration of these ideas, and the performance of the LIF scheme is critically examined. The chosen test-case is related to transient laminar convection within a parallel-plates channel. The developing thermal problem is solved for fully developed flow situation, considering axial diffusion in the energy equation, and the numerical challenging boundary condition of a step change in inlet temperature.

Solution methodology

As an illustration of the formal integral transform procedure, a transient convection-diffusion problem of a potential T (velocity, temperature or concentration) is considered. The problem is defined in the region V with boundary surface S and including nonlinear effects in the source terms as follows:

$$w(\mathbf{x})\frac{\partial T(\mathbf{x},t)}{\partial t} = \nabla\cdot k(\mathbf{x})\nabla T - d(\mathbf{x})T + P(\mathbf{x},t,T), \quad \text{in } \mathbf{x}\in V, \ \ t>0 \tag{1a}$$

with initial and boundary conditions:

$$T(\mathbf{x},0) = f(\mathbf{x}), \quad \mathbf{x}\in V \tag{1b}$$

$$\alpha(\mathbf{x})T + \beta(\mathbf{x})k(\mathbf{x})\frac{\partial T}{\partial \mathbf{n}} = \phi(\mathbf{x},t,T), \quad \mathbf{x}\in S, \ \ t>0 \tag{1c}$$

where α and β are the boundary condition coefficients and $\mathbf{n}$ is the outward drawn normal vector to surface S.

The equation and boundary source terms, P and ϕ, may include nonlinear operators from the original problem formulation, such as a nonlinear convection term, and therefore, the linear coefficients in system (1a), (1b) and (1c) can be interpreted as characteristic ones. Then, equations (1a), (1b) and (1c) can be viewed as a much more general formulation than apparent at first sight.

The first step in application of the GITT is the proposition of a filtering solution, which at least reduces the effects on convergence rates due to the equation and boundary source terms. The most frequently employed procedure is the adoption of a single analytical filter (Cotta, 1993; Cotta and Mikhailov, 1997), which in general reproduces the steady-state solution of the original problem or, at least a quasi-steady behavior upon linearization. The single filter strategy cannot offer an effective and uniform filtering over the whole time domain, and multiple successive filters may be required for further improvement of the final convergence rates. Since, the single filter is derived from a simplified problem formulation, in general linearizing or omiting terms in the original formulation, it will not follow the desired solution pattern for all the space and time domains ranges, which may result in quite variable magnitudes of the filtered source terms. More recently, a local-instantaneous filtering strategy was proposed, Macedo *et al.* (1999), which includes both space and time dependence, extracted from linearized and/or simplified versions of the original partial differential system. The scheme automatically updates the filter along the time integration march, and offers improved convergence rates enhancement, with respect to the single filtering strategy, acting on diminishing the importance of the source terms along the entire solution domain, in both time and space.

We then proceed with the derivation of an integral transform procedure combined with a general filtering obtained from any previously obtained approximate solution of the originally proposed partial differential system. This approximate solution is here denoted by $T_{\mathrm{F}}(\mathbf{x};t)$ and is considered a filtering solution in the form:

$$T(\mathbf{x},t) = T^*(\mathbf{x},t) + T_{\mathrm{F}}(\mathbf{x};t) \tag{2}$$

where $T_{\mathrm{F}}(\mathbf{x};t)$ has an analytical representation originated from the approximate solution methodology (or from discrete points interpolation), and may be defined in a specified range of the time variable only, $t_{s-1} < t < t_s$.

The resulting formulation for the filtered potential, T^*, then becomes:

$$w(\mathbf{x})\frac{\partial T^*(\mathbf{x},t)}{\partial t} = \nabla\cdot k(\mathbf{x})\nabla T^* - d(\mathbf{x})T^* + P^*(\mathbf{x},t,T^*), \quad \mathbf{x}\in V, \ \ t>0 \tag{3a}$$

with initial and boundary conditions:

$$T^*(\mathbf{x},0) = f^*(\mathbf{x}), \quad \mathbf{x}\in V \tag{3b}$$

$$\alpha(\mathbf{x})T^* + \beta(\mathbf{x})k(\mathbf{x})\frac{\partial T^*}{\partial \mathbf{n}} = \phi^*(\mathbf{x},t,T^*), \quad \mathbf{x}\in S \tag{3c}$$

where the filtered source terms and initial condition are given by:

$$P^*(\mathbf{x},t,T^*) = P(\mathbf{x},t,T) - \left[w(\mathbf{x})\frac{\partial T_{\mathrm{F}}(\mathbf{x},t)}{\partial t} - \nabla\cdot k(\mathbf{x})\nabla T_{\mathrm{F}} + d(\mathbf{x})T_{\mathrm{F}}\right] \tag{4a}$$

$$\phi^*(\mathbf{x}, t, T^*) = \phi(\mathbf{x}, t, T) - \left[\alpha(\mathbf{x})T_F + \beta(\mathbf{x})k(\mathbf{x})\frac{\partial T_F}{\partial \mathbf{n}}\right] \tag{4b}$$

$$f^*(\mathbf{x}) = f(\mathbf{x}) - T_F(\mathbf{x}; 0) \tag{4c}$$

In case, the approximate solution identically satisfies the initial and boundary conditions, such as in the situation of prescribed boundary potentials, the boundary source term and the initial distribution become zero, and only the filtered equation source term remains as defined in equation (4a). It is already evident that the time-variable filter strategy can possibly offer a more effective and uniform filtering over the whole time domain, and yield a fast converging eigenfunction expansion for the original problem. At this point, it suffices to proceed with the general filter strategy, and obtain the integral transform solution for the filtered potential, T^*. Following the formalism in the GITT (Cotta, 1993, 1994a, 1994b, 1998; Serfaty and Cotta, 1992; Cotta and Mikhailov, 1997), the appropriate eigenvalue problem is chosen as:

$$\nabla \cdot k(\mathbf{x})\nabla\psi_i(\mathbf{x}) + \left(\mu_i^2 w(\mathbf{x}) - d(\mathbf{x})\right)\psi_i(\mathbf{x}) = 0, \quad \mathbf{x} \in V \tag{5a}$$

with boundary conditions:

$$\alpha(\mathbf{x})\psi_i(\mathbf{x}) + \beta(\mathbf{x})k(\mathbf{x})\frac{\partial \psi_i}{\partial \mathbf{n}} = 0, \quad \mathbf{x} \in S \tag{5b}$$

and the solution for the associated eigenfunctions, $\psi_i(\mathbf{x})$, and eigenvalues, μ_i, is here assumed to be known. Problems (5a) and (5b) allow definition of the following integral transform pair:

$$\bar{T}_i(t) = \int_v w(\mathbf{x})\tilde{\psi}_i(\mathbf{x})T^*(\mathbf{x}, t)\mathrm{d}v, \quad \text{transform} \tag{6a}$$

$$T^*(\mathbf{x}, t) = \sum_{i=1}^{\infty} \tilde{\psi}_i(\mathbf{x})\bar{T}_i(t), \quad \text{inverse} \tag{6b}$$

where the normalized eigenfunctions are given by:

$$\tilde{\psi}_i(\mathbf{x}) = \frac{\psi_i(\mathbf{x})}{N_i^{1/2}} \tag{7}$$

and the normalization integrals:

$$N_i = \int_v w(\mathbf{x})\psi_i^2(\mathbf{x})\mathrm{d}v \tag{8}$$

After application of the integral transformation concept, the resulting ODE system for the transformed potentials, $\bar{T}_i(t)$, is written as:

$$\frac{\mathrm{d}\bar{T}_i(t)}{\mathrm{d}t} + \mu_i^2\bar{T}_i(t) = \bar{g}_i(t, \bar{T}_j), \quad t > 0, \ \ i,j = 1, 2\ldots \tag{9a}$$

with initial conditions:

$$\bar{T}_i(0) = \bar{f}_i \tag{9b}$$

where

$$\bar{g}_i(t, \bar{T}_j) = \int_v \tilde{\psi}_i(\mathbf{x}) P^*(\mathbf{x}, t, T^*) \mathrm{d}v + \int_S \phi^*(\mathbf{x}, t, T^*) \left(\frac{\tilde{\psi}_i(\mathbf{x}) - k(\mathbf{x}) \frac{\partial \tilde{\psi}_i}{\partial \mathbf{n}}}{\alpha(\mathbf{x}) + \beta(\mathbf{x})} \right) \mathrm{d}s \tag{9c}$$

$$\bar{f}_i = \int_v w(\mathbf{x}) \tilde{\psi}_i(\mathbf{x}) f^*(\mathbf{x}) \mathrm{d}v \tag{9d}$$

System (9a)-(9d) is then numerically solved through well-established initial value problem solvers, readily available in scientific subroutines libraries such as the IMSL Library (1989), or directly as built-in function in mixed symbolic-numerical platforms, such as the *Mathematica* system (Wolfram, 1996), which implement automatic relative error control schemes.

The desired final solution is then reconstructed by:

$$T(\mathbf{x}, t) = \sum_{i=1}^{N} \tilde{\psi}_i(\mathbf{x}) \bar{T}_i(t) + T_{\mathrm{F}}(\mathbf{x}; t) \tag{10}$$

The truncation order N may be adaptively chosen along the numerical integration march, as described in Cotta (1993), so as to always work with truncation orders that are just enough to satisfy the user prescribed accuracy requirements, at selected positions ($\mathbf{x}$) and time values (t).

A relative error estimator for the available hybrid solution may also be provided, at practically no additional cost, for a certain truncation order N in the eigenfunction expansion and evaluated at those points of interest, for instance by taking the last three terms in the expansion, in the form:

$$\varepsilon(\mathbf{x}, t) = \frac{\sum_{i=N-2}^{N} \tilde{\psi}_i(\mathbf{x}) \bar{T}_i(t)}{T(\mathbf{x}; t)} \tag{11}$$

An odd number of terms is in general preferred in the evaluation of the numerator of equation (11), so as to avoid eventual error cancellation due to oscillatory convergence patterns, with the associated false convergence indication. One should keep in mind that the above testing procedure, as an error estimator, relies on the approximate solution itself, and should be analyzed with care for very low truncation orders and/or slowly converging series. It is worth mentioning that the time-variable filtering strategy indirectly introduces a quite desirable modulation effect on the transformed ODE system. As the filter closely follows the desired solution functional behavior along the time domain, the differences in time constants among the filtered potentials are then reduced, i.e. the original transformed potentials obtained without filtering would certainly result in more widely spaced behaviors in the time variable among the solution components. While the single steady filter solution produces, in general, stiff ODE systems for increasing truncation orders, requiring special initial value problem solvers, the transient filtering shall yield, in principle, less stiff systems, which are readily solvable by standard explicit schemes at reduced computational cost. Of course, the degree of stiffness reduction offered by the employed LIF will depend on how closely the filter can follow the time variable behavior of the original function that is being pre-estimated by this strategy.

Application

The solution scheme here presented is now illustrated through consideration of transient laminar convection within a parallel-plates channel, according to the geometry and coordinates system shown in Figure 1. This hybrid numerical-analytical approach, especially useful in the development of benchmark solutions, is here analyzed in handling a test case in transient convection (Cotta *et al.*, 1986; Cotta and Gerk, 1994; Kakaç *et al.*, 1989; Cotta and Özisik, 1986; Kim *et al.*, 1990) within a considerable range of diffusion and convection relative influences. The solution strategy by analytical filtering with an approximate solution defined in certain time ranges, previously called a LIF (Macedo *et al.*, 1999), is undertaken to allow for a critical comparison of its relative efficiency with respect to previous filtering strategies (Gondim, 1997; Gondim and Cotta, 2000a, b).

The developing thermal problem is solved for fully developed flow situation, considering axial diffusion in the energy equation (Cotta *et al.*, 1986; Cotta and Gerk, 1994; Kakaç *et al.*, 1989), with step change of inlet temperature. An analysis is performed on the variation of Peclet number, so as to investigate the importance of the axial heat diffusion.

Considering the following dimensionless variables:

$$x = \frac{x^*/b}{Re\,Pr} = \frac{x^*}{b\,Pe}; y = \frac{y^*}{b}; u = \frac{u^*}{16u_{av}}; t = \frac{\alpha t^*}{b^2}; L = \frac{L^*/b}{Re\,Pr};$$

$$T = \frac{T^* - T_w}{T_e - T_w}; Re = \frac{u_{av}4b}{\nu}; Pr = \frac{\nu}{\alpha}; Pe = Re\,Pr = \frac{u_{av}4b}{\alpha} \quad (12)$$

the problem under concern is formulated in dimensionless form as:

$$\frac{\partial T(x,y,t)}{\partial t} + u(y)\frac{\partial T(x,y,t)}{\partial x} = \frac{\partial^2 T(x,y,t)}{\partial y^2} + \frac{1}{Pe^2}\frac{\partial^2 T(x,y,t)}{\partial x^2} \quad (13a)$$

$$0 < y < 1, \quad x > 0, \quad t > 0$$

$$T(x,y,0) = 0, \quad x \geq 0, \quad 0 \leq y \leq 1 \quad (13b)$$

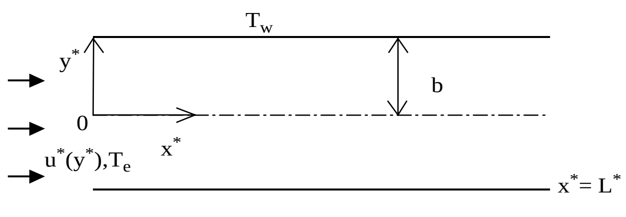


Figure 1. Coordinates system and problem geometry for transient convection between a parallel-plates channel with step change in inlet temperature

$$T(0, y, t) = 1, \quad t > 0, \ 0 \leq y \leq 1 \tag{13c}$$

$$\frac{\partial T(x, 0, t)}{\partial y} = 0, \quad t > 0, \ x \geq 0 \tag{13d}$$

$$T(x, 1, t) = 0, \quad t > 0, \ x \geq 0 \tag{13e}$$

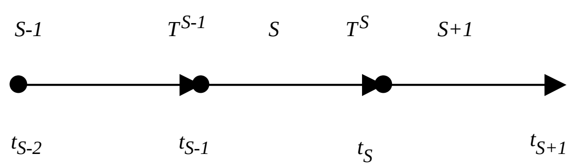
$$T(L, y, t) = 0, \quad t > 0, \ 0 \leq y \leq 1 \tag{13f}$$

with

$$u(y) = \frac{3}{32}(1 - y^2) \tag{13g}$$

This problem has been previously solved by employing a filtering solution for the steady-state and pure diffusion problem (Gondim, 1997; Gondim and Cotta, 2000b), and also validated via the reproduction of the infinite Peclet number results available in the literature (boundary layer formulation) (Cotta *et al.*, 1986; Cotta and Gerk, 1994). An approximate transient analytical solution has also been obtained for the same problem, through a combination of the methods GITT/Laplace transform (Cotta and Özisik, 1986; Kim *et al.*, 1990), which was then critically compared to the fully converged solution (Gondim, 1997; Gondim and Cotta, 2000b). This approximate analytical solution was also employed as a transient convective-diffusive filter in the complete solution of the original problem, with very good results, but with a marked increase in computational cost in comparison with the first alternative of a simple steady-state filter (Gondim, 1997; Gondim and Cotta, 2000a).

Aimed at reducing the computational cost, a local-instantaneous filtering strategy (Macedo *et al.*, 1999) is here applied to the original problem. This LIF, for each selected time interval, updates the information on the source terms in the approximate formulation of the filtering solution, so as to optimize the convergence rates along the time-integration process. Convergence is analyzed by increasing the number of terms in the related series and/or by increasing the number of intervals in the filter updating. Each time interval for the filter updating, $t_{S-1} < t < t_S$, has the following interpretation (Macedo *et al.*, 1999):



where S is the observed time interval, T^{S-1} is the temperature at the beginning of the process (refers to the previous interval) and t_{S-1} is the initial time. The desired potential $T(x, y, t)$ is then separated as:

$$T^s(x,y,t) = T^{*s}(x,y,t) + T^s_{\mathrm{f}}(x,y,t), \quad 0 \le y \le 1, \ x \ge 0, \ t_{s-1} \le t \le t_s \tag{14}$$

where $T^s_{\mathrm{f}}(x,y,t)$ is the LIF and $T^{*s}(x,y,t)$ is the filtered potential to be determined. The filter equation is chosen, from possible simplifications of the original problem, as:

$$\frac{\partial T^s_{\mathrm{f}}(x,y,t)}{\partial t} = \frac{\partial^2 T^s_{\mathrm{f}}(x,y,t)}{\partial y^2} + \frac{1}{Pe^2}\,\frac{\partial^2 T^s_{\mathrm{f}}(x,y,t)}{\partial x^2} + g^{s-1}(x,y), \tag{15a}$$

$$0 \le y \le 1, \ x \ge 0, \ t_{s-1} \le t \le t_s$$

where the simplified source term becomes:

$$g^{s-1}(x,y) = -u(y)\frac{\partial T^{s-1}(x,y,t_{s-1})}{\partial x} \tag{15b}$$

and the initial condition is updated accordingly:

$$T^s_{\mathrm{f}}(x,y,t_{s-1}) = T^{s-1}(x,y,t_{s-1});$$

$$\text{while for } t = 0, \ T^1_{\mathrm{f}}(x,y,0) = 0, \ x \ge 0, \ 0 \le y \le 1 \tag{15c}$$

A criterion of maximum relative deviation between the filter and the obtained solution may be employed for an automatic triggering of the filter updating, or more simply, an initial conservative estimate for the number of time intervals between updates may be obtained from the first time interval solution behavior.

Results and discussion

The computer code was prepared in Fortran (Microsoft Powerstation) and executed on a PC compatible microcomputer. The initial value problem for the transformed potentials was solved employing subroutine DIVPAG from the IMSL Library (1989). This subroutine can handle both non-stiff (Adams-Moulton) and stiff (Gear) systems, and their combined use is an interesting analysis tool in observing the ODE system degree of stiffness. The local relative error control was here prescribed from 10^{-4} to 10^{-8}, for checking purposes. The dimensionless times of interest selected are $t = 0.005$, 0.01, 0.03 e 0.05, where the first one is employed to observe the convergence behavior in the very low time limit, not in general recommended for eigenfunction expansion approaches, and the other three are selected to allow for comparisons with previously published results, as the Peclet number is varied. The selected Peclet number values are $Pe = 1$, 10 and 100, and a careful convergence analysis was performed for each of these situations, adopting a sufficiently large channel length in each case to warrant the application of the boundary condition at $x = L$.

Figure 2(a)-(c) shows the behavior of the dimensionless bulk temperature or concentration along the channel length, for the four dimensionless time values, and the different Peclet numbers in each case. Three sets of curves are shown in each graph, which correspond to the present GITT solution with a local instantaneous filter (GITT with LIF), the behavior of the filter itself (LIF only) as an approximate solution, and the previously obtained results for the integral transform approach with a single steady-state filter (Gondim, 1997; Gondim and Cotta, 2000b) for covalidation. One may observe the good agreement with the previous simpler GITT solution (Gondim and

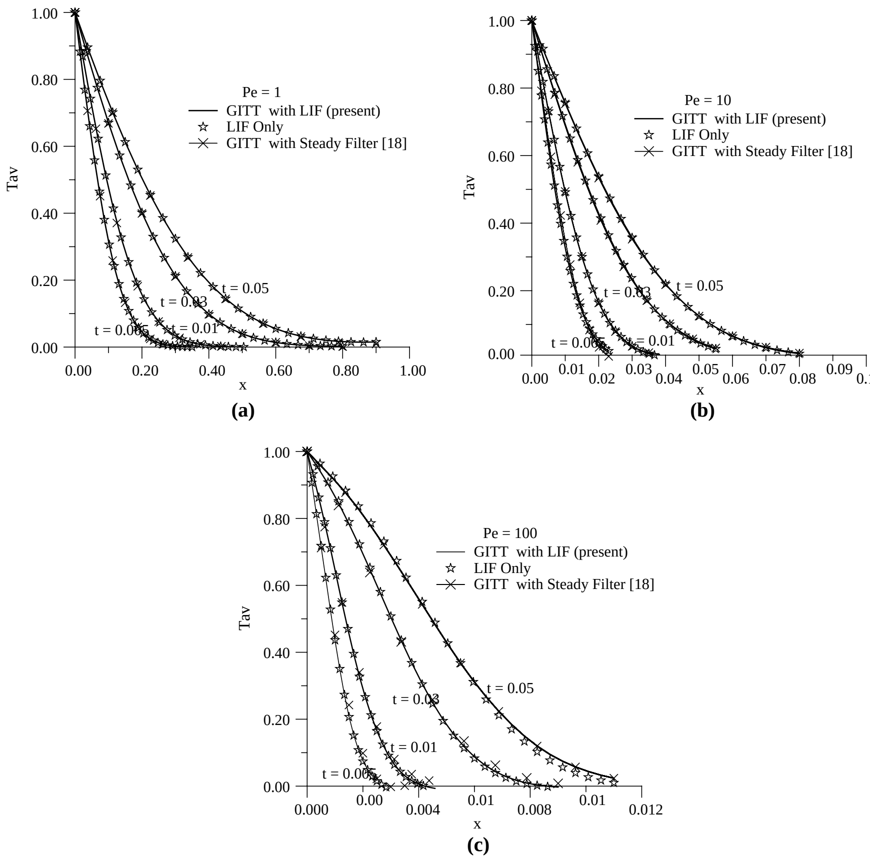


Figure 2.
(a) Comparison of solutions for the dimensionless bulk potential along the channel with $Pe = 1$ ($t = 0.005$, 0.01, 0.03 and 0.05); (b) comparison of solutions for the dimensionless bulk potential along the channel with $Pe = 10$ ($t = 0.005$, 0.01, 0.03 and 0.05); (c) comparison of solutions for the dimensionless bulk potential along the channel with $Pe = 100$ ($t = 0.005$, 0.01, 0.03 and 0.05)

Cotta, 2000b) with the more elaborate one here advanced. However, for the larger value of $Pe = 100$ it becomes evident that the GITT solution with the steady filter is yet not fully converged just after the thermal front region. In addition, the LIF solution itself may be considered as a reasonable approximation for the lower values of Pe, when diffusion is more important in the transport phenomena. The analysis also shows that one should not neglect, a priori, the axial diffusion in certain problems without a careful investigation that accounts for the effects of the boundary and inlet conditions employed in the formulation.

The convergence behavior of the present solution is illustrated in tabular form, according to Tables I and II for the dimensionless bulk potential, with $Pe = 10$ and 100, respectively. For the lower Peclet number, truncation orders around $N = 180$ were sufficient to warrant at least four significant digits of convergence, within the wide range of dimensionless times and axial positions here considered. For $Pe = 100$, larger truncation orders (up to $N = 240$) were required to reach convergence in general to the fourth significant digit of the bulk potential, while in the lower value of the time variable only three digits have stabilized at this system size. The consideration of a larger number of time steps in this region of lower time values can be employed, when

Time t	$\mathbf{X}$	Truncation order 100	120	140	160	180
0.005	0.0009583	0.92531	0.92511	0.92499	0.92499	0.92500
	0.0047917	0.64017	0.63940	0.63891	0.63891	0.63894
	0.0086250	0.39908	0.39842	0.39801	0.39797	0.39793
	0.0124583	0.22052	0.22052	0.22050	0.22042	0.22033
	0.0162917	0.10631	0.10689	0.10716	0.10709	0.10703
0.010	0.0016667	0.90807	0.90816	0.90831	0.90839	0.90845
	0.0083333	0.56635	0.56648	0.56667	0.56673	0.56677
	0.0937500	0.30063	0.30043	0.30003	0.29985	0.29972
	0.0216667	0.13206	0.13193	0.13188	0.13194	0.13202
	0.0283333	0.04742	0.04761	0.04793	0.04801	0.04801
0.030	0.0022917	0.92666	0.92673	0.92684	0.92690	0.92694
	0.0114583	0.65050	0.65051	0.65047	0.65043	0.65040
	0.0206250	0.41418	0.41409	0.41401	0.41402	0.41405
	0.0297917	0.23727	0.23737	0.23749	0.23749	0.23746
	0.0389583	0.12149	0.12139	0.12120	0.12117	0.12118
0.050	0.0033333	0.91609	0.91616	0.91628	0.91633	0.91637
	0.0166667	0.60716	0.60707	0.60693	0.60687	0.60684
	0.0300000	0.35680	0.35688	0.35696	0.35696	0.35694
	0.0433333	0.18348	0.18340	0.18338	0.18343	0.18347
	0.0566667	0.08148	0.08152	0.08144	0.08138	0.08137

Table I. Convergence behavior of dimensionless bulk potential, $T_{av}(\mathbf{x}, t)$, in various positions, $\mathbf{x}$, along the channel (LIF with three solution intervals) $Pe = 10$, $L = 0.2$

Time t	$\mathbf{X}$	Truncation order 120	160	200	220	240
0.005	0.0001667	0.90055	0.90302	0.90467	0.90527	0.90563
	0.0008333	0.52282	0.52548	0.52596	0.52600	0.52586
	0.0015000	0.21952	0.21306	0.20938	0.20859	0.20815
	0.0021667	0.04682	0.04665	0.04888	0.04971	0.05012
	0.0025000	0.00992	0.01426	0.01644	0.01653	0.01635
0.010	0.0002083	0.92815	0.93004	0.93110	0.93151	0.93172
	0.0010417	0.62965	0.63029	0.62981	0.62953	0.62933
	0.0018750	0.32992	0.32731	0.32700	0.32726	0.32738
	0.0027083	0.12379	0.12578	0.12578	0.12544	0.12517
	0.0035417	0.03033	0.02836	0.02764	0.02790	0.02809
0.030	0.0003750	0.95145	0.95248	0.95288	0.95313	0.95315
	0.0018750	0.71725	0.71677	0.71683	0.71699	0.71700
	0.0033750	0.43536	0.43505	0.43473	0.43475	0.43475
	0.0048750	0.19869	0.19918	0.19884	0.19873	0.19871
	0.0063750	0.06307	0.06289	0.06285	0.06267	0.06265
0.050	0.0004583	0.96167	0.96240	0.96257	0.96276	0.96274
	0.0022917	0.78256	0.78289	0.78306	0.78320	0.78319
	0.0041250	0.54913	0.54900	0.54914	0.54920	0.54919
	0.0059583	0.31208	0.31176	0.31187	0.31183	0.31183
	0.0077917	0.13578	0.13562	0.13573	0.13561	0.13561

Table II. Convergence behavior of dimensionless bulk potential, $T_{av}(\mathbf{x}, t)$, in various positions, $\mathbf{x}$, along the channel (LIF with three solution intervals) $Pe = 100, L = 0.02$

required, to enhance the filtering effect provided by the LIF and thus reduce the system truncation orders.

Finally, among the different possible filtering strategies studied for this class of problems, it can be concluded that the proposition of adaptive transient filtering, such as the LIF (Macedo *et al.*, 1999), is the most adequate alternative, due to the analytical and explicit tracking of the transient thermal front, where the effects of the non-transformable longitudinal convection term become relevant to the reduction of convergence rates.

Conclusion

In the present work, transient laminar convection within a parallel-plates channel is studied through the use of the GITT. The developing thermal problem is solved for fully developed flow situation, considering axial diffusion in the energy equation. An analysis is performed on the variation of Peclet number, so as to investigate the importance of the axial heat or mass diffusion. Aimed at reducing computational cost, a local-instantaneous filtering strategy is applied to the original problem, with excellent results, and convergence is analyzed by increasing the number of terms in the related series and/or by increasing the number of intervals in the filter updating. The developed code employs a general transient filtering solution originated from any approximate analytical solution of the original problem, and allows the reproduction of results available in the literature from different filtering strategies.

The analysis shows that one should not neglect a priori the axial diffusion in certain problems without a careful investigation that accounts for the boundary and inlet conditions employed in the formulation. It is also demonstrated that the use of local-instantaneous filters and dynamic reordering of the expansions is of crucial importance in the optimization of GITT solutions.

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